

Erratum

Numerical modelling of foam Couette flows

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Original article: Eur. Phys. J. E **27**, 123 (2008) DOI: 10.1140/epje/i2008-10358-7

Received: 3 February 2009

Published online: 14 March 2009 – © EDP Sciences / Società Italiana di Fisica / Springer-Verlag 2009

We have found a technical error in Appendix A of [1] which induced errors in Appendix B and in the figures. However, the text of the paper, including all conclusions drawn, is completely unaffected by the changes. In [1], equations (A.1a), (A.1b), and (A.1c) should read

$$\lambda \frac{\partial \tau_{rr}}{\partial t} + \max \left(0, 1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{rr} = 0, \quad (\text{A.1a})$$

$$\lambda \left(\frac{\partial \tau_{r\theta}}{\partial t} - 2\dot{\epsilon}_{r\theta} \tau_{rr} \right) + \max \left(0, 1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{r\theta} = 2\eta \dot{\epsilon}_{r\theta}, \quad (\text{A.1b})$$

$$\lambda \left(\frac{\partial \tau_{\theta\theta}}{\partial t} - 4\dot{\epsilon}_{r\theta} \tau_{r\theta} \right) + \max \left(0, 1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{\theta\theta} = 0, \quad (\text{A.1c})$$

with $\dot{\epsilon}_{r\theta} = 1/2 \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right)$. Consequently, Appendix B is modified but its conclusion remains the same: if the strain rate is discontinuous, we can predict the critical strain rate

$$\dot{\epsilon}_{r\theta}^c = \frac{1}{2\eta} \left[1 - \frac{\tau_Y}{\sqrt{2} |\tau_{r\theta}(r_c)|} \frac{1}{\left(1 + \frac{\lambda^2}{\eta^2} \tau_{r\theta}(r_c)^2 \right)^{1/2}} \right] \tau_{r\theta}(r_c).$$

All the presented computations have been made again,

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and most figures are unchanged, except Figures 7, 8(b), and 9(b), whose new versions are shown in the following page.

Modified Appendix B

[...] We write the constitutive equation with $\frac{\partial}{\partial t} = 0$. We have:

– When $r > r_c$: the plasticity term is zero, so (A.1c) leads to $2\lambda \dot{\epsilon}_{r\theta} \tau_{r\theta} = 0$, and $\dot{\epsilon}_{r\theta} = 0$ as $\tau_{r\theta} \neq 0$; in addition, we have $v = 0$ because of boundary conditions. However, (A.1a) and (A.1b) are then equivalent to $0 = 0$, and the normal stress components are not determined by the stationary equations only. In the transient problem, their values are determined by the initial conditions. Finally, as $v = 0$, (A.2b) yields $\tau_{r\theta} = C/r^2$, where C is a constant.

– When $r < r_c$: (A.1a, A.1b) lead to

$$\begin{aligned} \left(1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{rr} &= 0, \\ -2\lambda \dot{\epsilon}_{r\theta} \tau_{rr} + \left(1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{r\theta} &= 2\eta \dot{\epsilon}_{r\theta}, \\ -4\lambda \dot{\epsilon}_{r\theta} \tau_{r\theta} + \left(1 - \frac{\tau_Y}{|\boldsymbol{\tau}_d|} \right) \tau_{\theta\theta} &= 0. \end{aligned}$$

We denote with a^- (respectively, a^+) the quantities evaluated in $r = r_c^-$ (respectively in $r = r_c^+$); v and $\tau_{r\theta}$ are continuous, thus $v^- = v^+ = 0$, $\tau_{r\theta}^- = \tau_{r\theta}^+ = \tau_{r\theta}(r_c)$ and we

have

$$\begin{aligned} \left(1 - \frac{\tau_Y}{|\tau_d^-|}\right) \tau_{rr}^- &= 0, \\ -2\lambda \dot{\epsilon}_{r\theta}^- \tau_{rr}^- + \left(1 - \frac{\tau_Y}{|\tau_d^-|}\right) \tau_{r\theta}(r_c) &= 2\eta \dot{\epsilon}_{r\theta}^-, \\ -4\lambda \dot{\epsilon}_{r\theta}^- \tau_{r\theta}(r_c) + \left(1 - \frac{\tau_Y}{|\tau_d^-|}\right) \tau_{\theta\theta}^- &= 0. \end{aligned}$$

If $1 - \frac{\tau_Y}{|\tau_d^-|} = 0$, we find $\dot{\epsilon}_{r\theta}^- = 0 = \dot{\epsilon}_{r\theta}^+$: there is no discontinuity.

If $1 - \frac{\tau_Y}{|\tau_d^-|} \neq 0$, we find

$$\begin{aligned} \tau_{rr}^- &= 0, \\ \tau_{\theta\theta}^- &= 2\frac{\lambda}{\eta} \tau_{r\theta}(r_c)^2, \\ \dot{\epsilon}_{r\theta}^- &= \frac{1}{2\eta} \left(1 - \frac{\tau_Y}{|\tau_d^-|}\right) \tau_{r\theta}(r_c), \end{aligned}$$

with now

$$|\tau_d^-| = \left(2\tau_{r\theta}(r_c)^2 + 2\frac{\lambda^2}{\eta^2} \tau_{r\theta}(r_c)^4\right)^{1/2}.$$

As $\dot{\epsilon}_{r\theta}^- \neq \dot{\epsilon}_{r\theta}^+$, the strain rate is discontinuous at $r = r_c$ and we can define a critical strain rate

$$\dot{\epsilon}_{r\theta}^c = \frac{1}{2\eta} \left[1 - \frac{\tau_Y}{\sqrt{2}|\tau_{r\theta}(r_c)|} \frac{1}{\left(1 + \frac{\lambda^2}{\eta^2} \tau_{r\theta}(r_c)^2\right)^{1/2}} \right] \tau_{r\theta}(r_c).$$

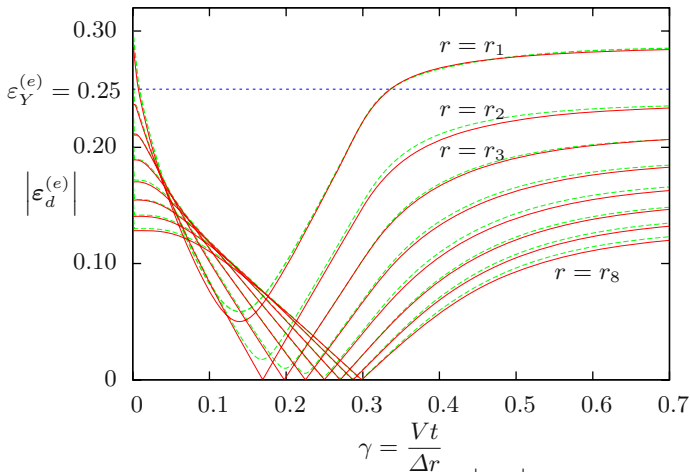


Fig. 7. (Colour online) Transient case: $|\epsilon_d^{(e)}|$ versus t for r from $r = r_1$ to r_8 . Dashed green lines: former computations; solid red lines: present computations.

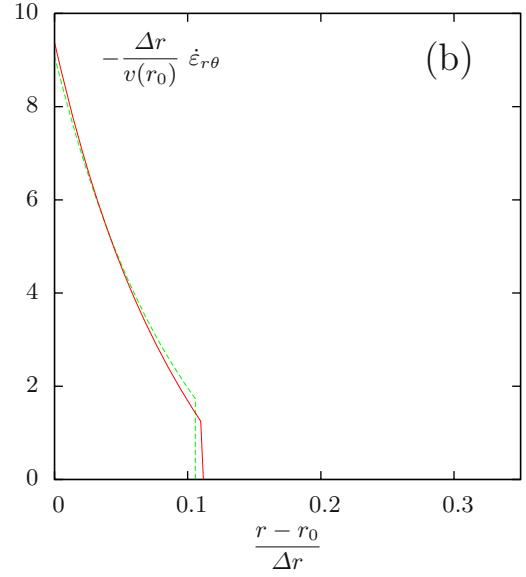


Fig. 8. (Colour online) Stationary case: (b) Shear strain rate $\dot{\epsilon}_{r\theta}$ versus r . There is no experimental data available for the comparison. Dashed green lines: former computations; solid red lines: present computations.

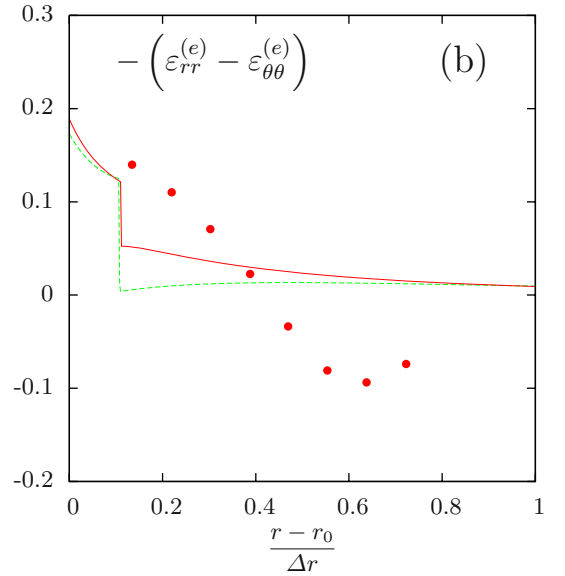


Fig. 9. (Colour online) Stationary case: (b) Difference of normal components. $-(\epsilon_{rr}^{(e)} - \epsilon_{\theta\theta}^{(e)})$ versus r . Dashed green lines: former computations; solid red lines: present computations; \bullet : experimental data.

References

1. I. Cheddadi, P. Saramito, C. Raufaste, P. Marmottant, F. Graner, *Eur. Phys. J. E* **27**, 123 (2008).